WEEKLY TEST MEDICAL PLUS -03 TEST - 11 RAJPUR SOLUTION Date 06-10-2019

[PHYSICS]

1. **(b)** Moment of inertia ($= \sum mr^2$) for a given body depends on the axis of rotation, mass, shape and size of the body as well as on the distribution of mass within the body. Farther the constituent particles of a body are from the axis of rotation, larger will be its moment of inertia. So moment of inertia does not depend only on angular velocity.

2. **(a)** Let C be the centre of mass of the dumb-bell and the particles of masses m_1 and m_2 be placed at distances r_1 and r_2 from C. Hence, moment of inertia of given system about an axis passing through CM,
$I = m_1r^2 + m_2r_2^2$
According to definition of the centre of mass,
$m_1r_1 = m_2r_2$
Also, $r_1 + r_2 = r$
Solving for r_1 and r_2 above eqn.

$$r_1 = \frac{m_2 r}{(m_1 + m_2)} \text{ and } r_2 = \frac{m_1 r}{(m_1 + m_2)}$$

$$\therefore\ I = \frac{m_1 m_2 r^2}{(m_1 + m_2)}$$

3. **(c)** According to parallel axes theorem,

$$I = I_{CG} + Md^2 = \frac{Ml^2}{12} + Md^2$$

$$= 300\left[\frac{100^2}{12} + 20^2\right] = 3.7 \times 10^5 \text{ gm-cm}^2$$

4. B

5. **(c)** $I_{COD} = \sum mr^2 = m_A r_A^2 + m_B r_B^2$

$$= ma^2 + m\,(a\cos 60°)^2$$

$$= ma^2 + m\frac{a^2}{4} = \frac{5}{4}ma^2$$

6. **(a)** $I_z = I_1 + I_2 + I_3$

$$= \frac{ML^2}{3} + \frac{ML^2}{3} + 0 = \frac{2ML^2}{3}$$

7. **(a)** According to perpendicular axis theorem,
I_{EF} = MI of system about one rod as axis + MI of system about second rod as axis

$$= \frac{ML^2}{12} + \frac{ML^2}{12} = \frac{ML^2}{6}$$

8. **(d)** $I_{\text{median line}} = I_A + I_B + I_C + I_D$

$$= 2 \times \frac{Ml^2}{12} + 2M\left(\frac{l}{2}\right)^2 = \frac{Ml^2}{6} + \frac{Ml^2}{2} = \frac{2}{3}Ml^2$$

9. **(a)** Diagonals are also mutually ⊥. Hence,

$$I_D + I_D = \frac{4}{3}Ml^2 \qquad \therefore \quad I = \frac{2}{3}Ml^2$$

10. **(a)** $I = I_1 + I_1 = 2 \times \frac{2}{3}Ml^2 = \frac{3}{4}Ml^2$

11. **(d)** $I = I_A + I_B + I_C = 0 + \frac{Ml^2}{3} + Ml^2 = \frac{4}{3}Ml^2$

12. **(b)** $I_x = I_y = 2\left[\frac{ml^2}{3}\sin^2 45°\right] = \frac{ml^2}{3}$

$$I_z = 2\left[\frac{ml^2}{3}\right] = \frac{2}{3}ml^2 \;\; \therefore \; I_x = I_y < I_z$$

13. **(a)** The desired moment of inertia is,

$$I = \int_{x=-l}^{x=+l} dI = \int_{-l}^{+l}\left(\frac{m}{2l}dx\right)(x\sin\alpha)^2 = \frac{ml^2}{3}\sin^2\alpha$$

14. **(c)** As $I_z = 2I$ where, $I = \frac{MR^2}{4}$

According to parallel axes theorem, required moment of inertia about axis TT' is
$I'_{TT} = I_z + MR^2 = 2I + MR^2 = 2I + 4I = 6I$

15. **(b)** Moment of inertia depends on the distribution of mass around the axis. Farther the constituent particles of a body from the axis of rotation, larger will be its moment of inertia.

16. **(c)** As the planes of two rings are mutually ⊥ and the centres are coincident, hence an axis, which is passing through the centre of one of the rings and ⊥ to its plane, will be along the diameter of other ring. Hence, moment of inertia of the system

$$= I_{CM} + I_{\text{diameter}} = mr^2 + \frac{mr^2}{2} = \frac{3}{2}mr^2$$

17. **(a)**
The moment of inertia of a flywheel is given by:
$I = MR^2$
Taking log on both sides,
$\log I = \log M + 2 \log R$
Differentiating it, we get;

$$\frac{dI}{I} = 0 + 2\frac{dR}{R}$$

$$100 \times \frac{dI}{I} = 2\left[\frac{dR}{R} \times 100\right]$$

$\therefore$ % increase in moment of inertia = 2 × 1 = 2%

18. **(c)** Let M be the mass of each disc. Let R_A and R_B be the radii of discs A and B respectively. Then

$$M = \pi R_A^2 t d_A = \pi R_B^2 t d_B$$

As, $d_A > d_B \therefore R_A^2 < R_B^2$

Now, $I_A = \frac{1}{2}MR_A^2, I_B = \frac{1}{2}MR_B^2$

$$\therefore \frac{I_A}{I_B} = \frac{R_A^2}{R_B^2} < 1 \text{ i.e., } I_A < I_B$$

19. **(b)** Suppose, M be the mass of the annular disc of outer radius R and inner radius r.
Then surface mass density $= \sigma = \frac{\text{mass}}{\text{area}}$

$$= \frac{M}{\pi(R^2 - r^2)}$$

Mass of elementary ring of radius x and thickness dx

$$= \frac{M}{\pi(R^2 - r^2)} \times 2\pi x \ dx = \frac{2Mx\,dx}{(R^2 - r^2)}$$

MI of ring about an axis passing through the centre of mass and perpendicular to its plane is

$$dI = \frac{2Mx\,dx}{(R^2 - r^2)}x^2 = \frac{2Mx^3\,dx}{(R^2 - r^2)}$$

$$\therefore I = \int_r^R \frac{2Mx^3\,dx}{(R^2 - r^2)} = \frac{2M}{(R^2 - r^2)}\left[\frac{R^4 - r^4}{r}\right]$$

$$= \frac{1}{2}M(R^2 + r^2)$$

20. **(d)** Mass of disc ∝ area

$$\therefore M_A = 4M_B \therefore \frac{I_A}{I_B} = \frac{\frac{1}{2}M_A R_A^2}{\frac{1}{2}M_B R_B^2} = 4\times 4 = 16$$

21. **(d)** Moment of inertia of discs A and B about the axis through their centre of mass and perpendicular to the plane will be,

$$I_{AA} = I_{BB} = \frac{1}{2}Mr^2$$

Now, moment of inertia of disc A about an axis through B, by theorem of parallel axes will be,

$$I_{AB} = I_{AA} + M(2r)^2 = \frac{1}{2}Mr^2 + 4Mr^2 = \frac{9}{2}Mr^2$$

So,

$$I = I_{BB} + I_{AB} = \frac{1}{2}Mr^2 + \frac{9}{2}Mr^2 = 5Mr^2$$

22. **(d)** $I = \frac{2\pi R}{4}$ or $R = \frac{2l}{\pi}$

$$\therefore I = mR^2 = mm\left(\frac{2l}{\pi}\right)^2 = 0.4ml^2 \text{ (as } \pi^2 \approx 10)$$

23. B

24. **(c)** M = Mass of the square plate before cutting the holes

Mass of one hole,

$$m = \left(\frac{M}{16R^2}\right)\pi R^2 = \frac{\pi M}{16}$$

∴ Moment of inertia of the remaining portion,

$$I = I_{\text{square}} - 4I_{\text{hole}}$$

$$= \frac{M}{12}(16R^2 + 16R^2) - 4\left[\frac{mR^2}{2} + m(2R^2)\right]$$

$$= \frac{8}{3}MR^2 - 10mR^2 = \left(\frac{8}{3} - \frac{10\pi}{16}\right)MR^2$$

25. **(c)** We know that; $\vec{L} = I\vec{\omega}$

$$\therefore \frac{dL}{dt} = I\frac{d\vec{\omega}}{dt} = I\vec{\alpha} \Rightarrow \frac{d\vec{L}}{dt} = \vec{\tau} \quad (\because \vec{\tau} = I\vec{\alpha})$$

If $\vec{\tau} = 0$, then $\frac{d\vec{L}}{dt} = 0$ *i.e.,* $\vec{L} =$ constant vector

26. C

27. **(b)** Suppose, when the beads reach the ends, moment of inertia of the system changes from I_1 to I_2 and therefore, the angular velocity changes from ω_1 to ω_2. As external torque acting on the system is zero, hence angular momentum of the system is conserved,

28. **(a)** When sand is poured on a rotating disc its moment of inertia gets increased. Since, no external torque acts on the system, hence angular momentum must be conserved. As $L = I\omega$, hence angular velocity will decrease.

29. D

30. **(c)** Applying conservation of angular momentum.

$$I_1\omega = (I_1 + I_2)\omega_1$$

$$\Rightarrow \quad \omega_1 = \frac{I_1}{(I_1 + I_2)}\omega$$

31. **(b)** $R_{C.M.} = \dfrac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$ (i)

After changing a position of m_1 and to keep the position of C.M. same

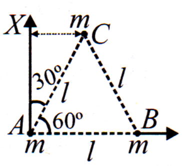
$$R_{C.M.} = \frac{m_1(r_1 - d) + m_2(r_2 + d_2)}{m_1 + m_2}$$

$$\Rightarrow \quad 0 = \frac{m_1 d + m_2 d_2}{m_1 + m_2}$$

[Substituting value of C.M. from Eq. (i)]

$$\Rightarrow \quad d_2 = \frac{m_1 d}{m_2}$$

32. **(c)** The moment of inertia of the system



$= m_A r_A^2 + m_B r_B^2 + m_C r_C^2$

$= m_A (0)^2 + m\,(l)^2 + m\,(l\sin 30°)^2$

$= ml^2 + ml^2 \times (1/4)$

$= (5/4)\,ml^2$

33. **(c)** $\text{K.E.} = \dfrac{1}{2}I\omega^2$

$$\therefore \quad \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}.2I_1\omega_2^2$$

$$\Rightarrow \quad \frac{\omega_1^2}{\omega_2^2} = \frac{2}{1} \qquad \Rightarrow \quad \frac{\omega_1}{\omega_2} = \frac{\sqrt{2}}{1}$$

34. **(c)** M.I. of disc about its normal

$= \frac{1}{2} MR^2$

M.I about its one edge $= MR^2 + \frac{MR^2}{2}$

(Perpendicular to the plane)

Moment of inertia $= \frac{3}{2} MR^2$

35. **(d)** Moment of inertia of a uniform circular disc about an axis through its centre and perpendicular to its plane is $I_C = \frac{1}{2} MR^2$

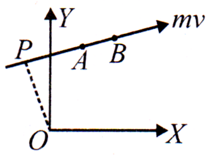
By the theorem of parallel axes,

∴ Moment of inertia of a uniform circular disc about an axis touching the disc at its diameter and normal to the disc is I.

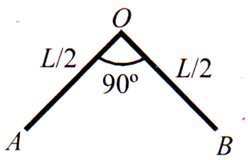
$$I = \frac{1}{2} MR^2 + MR^2 = \frac{3}{2} MR^2$$

36. **(a)** Moment of momentum is angular momentum. OP is the same whether the mass is at A or B

$\therefore L_A = L_B$.



37. **(d)** Total mass = M, total length = L



Moment of inertia of OA = OB about O

⇒ M.I. total

$$= 2 \times \left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 \cdot \frac{1}{3} = \frac{ML^2}{12}$$

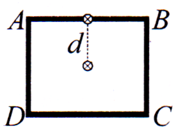
38. **(d)** M.I. of a circular disc, $Mk^2 = \frac{MR^2}{2}$

M.I. of a circular ring = MR^2

∴ Ratio of their radius of gyration

$= \frac{1}{\sqrt{2}} : 1$ or $1 : \sqrt{2}$

39. **(d)** Moment of inertia for the rod AB rotating about an axis through



40. **(d)** As $\tau^{ext} = 0$, hence $L_i = L_f$

According to question,

$$I_t\omega_i = (I_t + I_b)\omega_f \text{ or } \omega_f = \frac{I_t\omega_i}{(I_t + I_b)}$$

Initial energy, $E_i = \frac{1}{2}I_t\omega_i^2$

Final energy

$$E_f = \frac{1}{2}(I_t + I_b)\omega_f^2 = \frac{1}{2}(I_t + I_b)\left(\frac{I_t\omega_i}{I_t + I_b}\right)^2$$

$$= \frac{1}{2}\frac{I_t^2\omega_i^2}{(I_t + I_b)}$$

Loss of energy, $\Delta E = E_i - E_f$

$$= \frac{1}{2}I_t\omega_i^2 - \frac{1}{2}\frac{I_t^2\omega_i^2}{(I_t + I_b)} = \frac{\omega_i^2}{2}\left(I_t - \frac{I_t^2}{(I_t + I_b)}\right)$$

$$= \frac{\omega_i^2}{2}\left(\frac{I_t^2 + I_bI_t - I_t^2}{(I_t + I_b)}\right) = \frac{1}{2}\frac{I_bI_t}{(I_t + I_b)}\omega_i^2$$

41. **(a)** Mass of the disc = $9M$

Mass of removed portion of disc = M

The moment of inertia of the complete disc about an axis passing through its centre O and perpendicular to its plane is $I_1 = \frac{9}{2}MR^2$

Now, the moment of inertia of the disc with removed portion is $I_2 = \frac{1}{2}M\left(\frac{R}{3}\right)^2 = \frac{1}{18}MR^2$

Therefore, moment of inertia of the remaining portion of disc about O is

$$I = I_1 - I_2 = 9\frac{MR^2}{2} - \frac{MR^2}{18} = \frac{40MR^2}{9}$$

42. **(b)** When a mass is rotating in a plane about a fixed point its angular momentum is directed along a line perpendicular to the plane of rotation.

43. **(a)** According to the theorem of parallel axes,

$I = I_{CM} + Ma^2$

As a is maximum for point B.

Therefore I is maximum about B.

44. A
45. D

[CHEMISTRY]

46. (*c*) : Rate $= -\dfrac{d[N_2O_5]}{dt} = +\dfrac{1}{2}\dfrac{d[NO_2]}{dt}$

$= 2\dfrac{d[O_2]}{dt}$

Given $\dfrac{-d[N_2O_5]}{dt} = 6{\cdot}25 \times 10^{-3}$ mol L^{-1} s^{-1}

Rate of formation of NO_2

$= \dfrac{[NO_2]}{dt} = -2\dfrac{d[N_2O_5]}{dt}$

$= 2 \times 6{\cdot}25 \times 10^{-3}$ mol L^{-1} s^{-1}

$= 12{\cdot}50 \times 10^{-3}$ mol L^{-1} s^{-1}

$= 1{\cdot}25 \times 10^{-2}$ mol L^{-1} s^{-1}

Rate of formation of O_2

$= \dfrac{d[O_2]}{dt} = -\dfrac{1}{2}\dfrac{d[N_2O_5]}{dt}$

$= \dfrac{1}{2} \times 6{\cdot}25 \times 10^{-3}$ mol L^{-1} s^{-1}

$= 3{\cdot}125 \times 10^{-3}$ mol L^{-1} s^{-1}

47. (*b*) : Minus signs are for reactants and positive signs for products. Dividing numbers are the coefficients.

48. (*b*) : $A_2(g) \longrightarrow B(g) + \frac{1}{2}C(g)$

100	0	0
$100 - p$	p	$\frac{1}{2}p$

$100 - p + p + \frac{1}{2}p = 120$ or $p = 40$ mm

$\therefore -\dfrac{dp_{A_2}}{dt} = \dfrac{40}{5} = 8$ mm min^{-1}

49. (*c*) : Initially, Rate $= k[Y][Z]^{1/2}$

New rate $= k[Y][2Z]^{1/2}$

$= \sqrt{2}\,k[Y][Z]^{1/2} = 1{\cdot}414\,k[Y][Z]^{1/2}$.

50. (*d*) : The rate of reaction is same as expressed in terms of any reactant or product.

51. (*a*) : Rate of reaction $= \dfrac{1}{4}\dfrac{\Delta[NO_2]}{\Delta t}$

$= \dfrac{1}{4} \times \dfrac{[5{\cdot}2 \times 10^{-3}\,M]}{100\,s} = 1{\cdot}3 \times 10^{-5}$ M s^{-1}.

52. (*b*) : Rate $= k[NO]^2[O_2]$. Initially rate $= ka^2b$. If volume is reduced to half, concentration are doubled so that new rate

$= k(2a)^2(2b) = 8ka^2b$, *i.e.*, 8 times.

53. (c) : From slow step, rate = $k\,[B_2]\,[A]$.

From 1st eqn, $K_{eq} = \frac{[A]^2}{[A_2]}$

or $[A] = \sqrt{K_{eq}[A_2]} = K_{eq}^{1/2}[A_2]^{1/2}$

Hence, rate $= k\,[B_2]\,K_{eq}^{1/2}[A_2]^{1/2}$

$= k'\,[A_2]^{1/2}\,[B_2]$.

Hence, order $= 1\frac{1}{2}$.

54. (c) : On the basis of given units of k, the reaction is of 3rd order.

55. (d) : $r = k\,[A]^{\alpha}\,[B]^{\beta} = k\,a^{\alpha}\,b^{\beta}$. If concentration of B is doubled, $\frac{r}{4} = k\,a^{\alpha}\,(2\,b)^{\beta}$. Dividing 2nd eqn. by 1st eqn.,

$\frac{1}{4} = 2^{\beta}$ or $2\beta = 2^{-2}$. Hence, $\beta = -2$.

56. (a) : As step I is the slowest, hence it is the rate determining step.

57.

58. (d) : $k = \frac{2.303}{32}\log\frac{a}{a-0.99a}$

$= \frac{2.303}{32}\log 10^2 = \frac{2.303}{16}\ \text{min}^{-1}$

$t_{99.9\%} = \frac{2.303}{k}\log\frac{a}{a-0.999a}$

$= \frac{2.303}{k}\log 10^3 = \frac{3\times 2.303}{2.303}\times 16$

$= 48$ min.

59. (b) : 0.08 mol L^{-1} to 0.01 mol L^{-1} involves 3 half-lives.

60. (d) : $k = \frac{2.303}{t}\log\frac{a}{a-x}$

or $\log\frac{a}{a-x} = \frac{kt}{2.303} = \frac{2.2\times 10^{-5}\times 60\times 90}{2.303}$

$= 0.0516$.

Hence, $\frac{a}{a-x}$ = antilog 0.0516 = 1.127.

or $\frac{a-x}{a} = 0.887$ or $1-\frac{x}{a} = 0.887$

or $\frac{x}{a} = 0.113 = 11.3\%$.

61. (c) : $t_{90\%} = \frac{2.303}{k} \log \frac{a}{a - 0.9a}$

$= \frac{2.303}{k} \log 10 = \frac{2.303}{k}$

$t_{1/2} = \frac{2.303}{k} \log \frac{2}{a - a/2}$

$= \frac{2.303}{k} \log 2 = \frac{2.303}{k} \times 0.3010$

$\therefore\ t_{90\%}/t_{1/2} = \frac{1}{0.3010} = 3.3$

i.e., $t_{90\%} = 3.3$ times $t_{1/2}$.

62. (a) : Decrease in concentration from 0.8 M to 0.4 M in 15 minutes means $t_{1/2} = 15$ minutes. Time taken for decrease in concentration from 0.1 M to 0.25 M means two half-lives, *i.e.,* = 2 × 15 min = 30 min.

63. (c) : At the point of intersection, $[A] = [B]$, *i.e.,* half of the reactant has reacted. Hence, it represents $t_{1/2}$.

64 (a) : It P_t is the pressure after time t,

$$k = \frac{2.303}{t} \log \frac{P_0}{P_t}$$

$\therefore\ 3.38 \times 10^{-5}\ s^{-1} = \frac{2.303}{600\,s} \log \frac{500 \text{ atm}}{P_t}$

or $\log \frac{500}{P_t} = 0.0088$ or $\frac{500}{P_t} = 1.021$

or $P_t = 490$ atm

65. (a) : $[A]$ is kept constant, $[B]$ is doubled, rate is doubled. So rate $\propto [B]$.
$[B]$ is kept constant, $[A]$ is tripled, rate becomes 9 times, so rate $\propto [A]^2$.
Hence, rate law is

$$\text{rate} = k\,[A]^2\,[B]$$

66 (c) : The reaction occurring in two steps has two activation energy peaks.
The first step, being fast needs less activation energy. The second step, being slow, needs more activation energy. Therefore, second peak will be higher than the first.

67. (b) : Rate = $k\,[NOBr_2]\,[NO]$

From step I, $K_{eq} = \frac{[NOBr_2]}{[NO]\,[Br_3]}$

or $[NOBr_2] = K_{eq}\,[NO]\,[Br_2]$

Substituting in eqn. (*i*), we get

Rate = $k\,K_{eq}\,[NO]^2\,[Br_2] = k'\,[NO]^2\,[Br_2]$

Hence, order with respect to NO is 2.

68. (*b*) : $r = K[A]^{\alpha} = k a^{\alpha}$

$1.837\, r = k(1.5\, a)^{\alpha}$

Dividing, $1.837 = (1.5)^{\alpha}$

On solving, we get $\alpha = 1.5$

Hence order = 1·5

69. (*a*) : Half-life of a frist order reaction does not depend upon initial concentration. It is equal to ln 2/*k*.